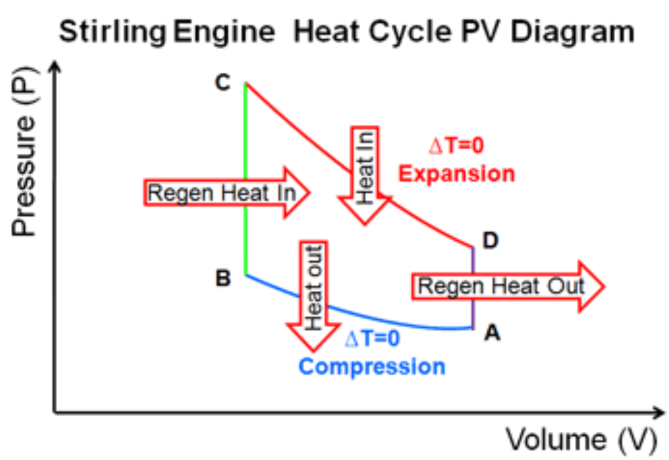
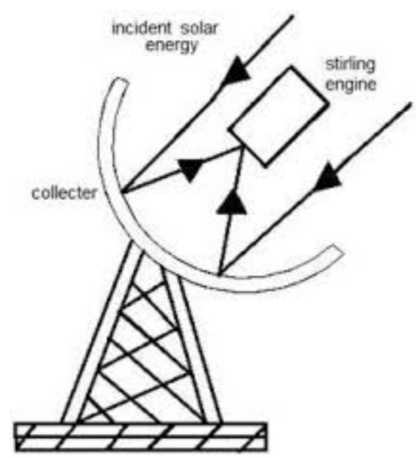
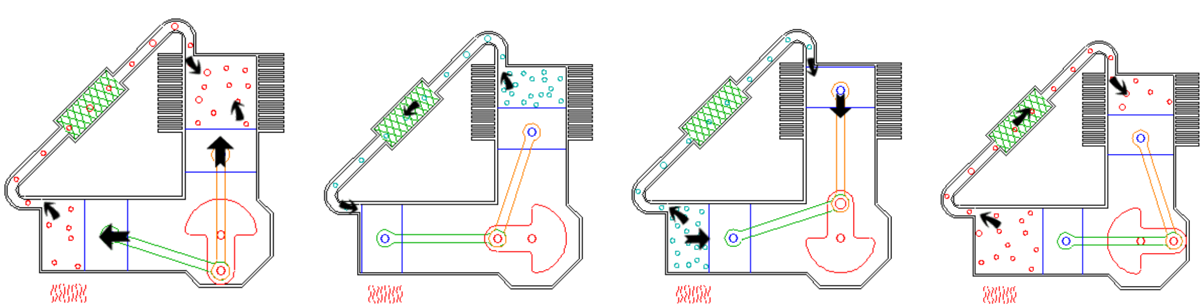
Chemical Engineering Thermodynamics
Quiz 2
January 24, 2019

A Sterling engine is a simple heat engine used to convert low grade heat such as process waste heat or solar heat to shaft work. It has been proposed as an alternative to photovoltaics and has found use in some applications such as pumping water. For harvesting solar energy, the moving parts, relatively low efficiency, and high capital cost of Sterling engine solar generators make them less desirable compared to cheap silicon photovoltaics from China.

The Sterling engine consists of two cylinders and a regenerator which is a heat exchanger that stores and transfers hot or cold energy between cycle steps acting as a preheater or a precooler for the next cycle step. Consider a Sterling engine using hydrogen gas as a working fluid and operating from 2MPa to 16 MPa with a temperature range of 40°C (A-B) to 300°C (C-D). Assume the ideal gas law is appropriate and $C_p = 7/2\ R$.







A-B Isothermal Compression

B-C Isochoric Regeneration or Transfer

C-D Isothermal Expansion

D-A Isochoric Regeneration or Transfer

T °C	**40**	**40-300**	**300**	**300-40**
P MPa	**2-5**	**5-16**	**16-8**	**8-5**

a) For the stage A-B and C-D calculate the work, W_{EC}, and Q.
b) Calculate the work, W_{EC}, and Q for stages B-C and D-A.
c) Calculate the overall work.
d) Consider smaller regeneration steps, so that the low pressures are increased. Explain how this would impact the overall work? Comment on the importance of the regenerative step in the Sterling engine. (Use the PV plot to visualize this change)

For your answers make a table of the type (or use this table):

	isothermal	isochoric	isothermal	isochoric	
	Stage A-B	**Stage B-C**	**Stage C-D**	**Stage D-A**	**Net**
Ti °K					x
Tf °K					x
Pi Mpa					x
Pf Mpa					x
W_{EC} kJ/mole					
ΔH kJ/mole					
ΔU kJ/mole					
Q kJ/mole					

1 atmosphere is 14.7 psi, 1.01 bar, 0.101 MPa, 760 mmHg, 29.9 inHg

Gas Constant, R

= 8.31447 J/mole-K = 8.31447 cm^3-MPa/mole-K = 8.31447 m^3-Pa/mole-K
= 8,314.47 cm^3-kPa/mole-K = 83.1447 cm^3-bar/mole-K = 1.9859 Btu/lbmole-R(see note 1)
= 82.057 cm^3-atm/mole-K = 1.9872 cal/mole-K(see note 2) = 10.731 ft^3-psia/lbmole-R

Process Type	Work Formula (ig)
Isothermal	$W_{EC} = -\int P dV = -RT\int \frac{dV}{V} = -RT\ln\frac{V_2}{V_1}$ (ig)
Isobaric	$W_{EC} = -\int P dV = -P(V_2 - V_1)$ (ig)
Adiabatic and reversible	$W_{EC} = -\int P dV = -\int \text{const}\frac{dV}{V^{(C_P/C_V)}}$ (*ig) or $\Delta U = C_V(T_2 - T_1) = W_{EC}$ (*ig) $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(R/C_P)} = \left(\frac{V_1}{V_2}\right)^{(R/C_V)}$ (*ig)

$Q_{rev} = \Delta U$ for isochoric (constant volume) 4.17

$dU = C_v\, dT$ for isochoric (constant volume)

$C_p = C_v + R$ (exact for ideal gas)

$\Delta H = \Delta U + \Delta(PV) = \Delta U + R(\Delta T)$ (exact for ideal gas)

ANSWERS: **Chemical Engineering Thermodynamics**

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	isothermal	*isochoric*	*isothermal*	*isochoric*	
	Stage A-B	**Stage B-C**	**Stage C-D**	**Stage D-A**	**Net**
Ti °K	313	313	573	573	x
Tf °K	313	575	573	313	x
Pi Mpa	2	5	16	8	x
Pf Mpa	5	16	8	5	x
W_{EC} kJ/mole	**-2.38**	**0**	**3.30**	**0**	**0.92**
ΔH kJ/mole	0	7.56	0	-7.56	0
ΔU kJ/mole	0	5.40	0	-5.40	0
Q kJ/mole	**2.38**	**5.40**	**-3.30**	**-5.40**	**-0.92**

(1)

a) stage A-B

isothermal 40°C (313°K)

$P_i = 2\ MPa \quad P_f = 5\ MPa$

$$W_{EC_{AB}} = -RT \ln \frac{V_2}{V_1} = -RT \ln \frac{P_1}{P_2}$$

$$= -8.31 \frac{MPa\ cm^3}{°K\ mole} (313°K) \ln \frac{2\ MPa}{5\ MPa}$$

$$W_{EC_{AB}} = -2.38\ kJ/mole$$

$\Delta H = \Delta U = 0 \quad \Delta T = 0$

$Q = -W_{EC} = 2.38\ kJ/mole$

stage C-D

isothermal 300°C (573°K)

$P_i = 16\ MPa \quad P_f = 8\ MPa$

$$W_{EC_{CD}} = 8.31 \frac{J}{°K\ mol} (573°K) \ln \frac{16\ MPa}{8\ MPa}$$

$= 3.30\ kJ/mole \qquad Q = -W_{EC} = -3.30\ kJ/mole$

(2)

b) ~~Stage~~ Stage B–C

isochoric $W_{EC_{BC}} = 0$

$P_i = 5\text{ MPa}$ $P_f = 16\text{ MPa}$

$T_i = 40°C\ (313°K)$ $T_f = 300°C\ (573°K)$

Isochoric $Q = \Delta U = \frac{5}{2}\ 8.31\ \frac{J}{mol\ °K}\ (573°K - 313°K)$

$= 5.40\ \frac{kJ}{mole}$

Ideal Gas diatomic $\Delta H = \frac{7}{5}(5.40\ \frac{kJ}{mole}) = 7.56\ \frac{kJ}{mole}$

Stage D–A $W_{EC_{DA}} = 0$

isochoric

$P_i = 8\text{ MPa}$ $P_f = 5\text{ MPa}$

$T_i = 300°C\ (573°K)$ $T_f = 40°C\ (313°K)$

Isochoric Const. Volume $Q = \Delta U = \frac{5}{2}(8.31\ \frac{J}{mole})(313°K - 573°K)$

$= -5.40\ kJ/mole$

ideal gas diatomic $\Delta H = \frac{7}{5}\ \Delta U = -7.56\ kJ/mole$

③

c) Overall Work

$$3.30 \frac{kJ}{mol} - 2.38 \frac{kJ}{mole} = 0.92 \frac{kJ}{mole}$$

d) Smaller regeneration steps would decrease the difference between the work in AB and DC cycles leading to less net work. The regeneration step is very important to an efficient Stirling process.